PURDUE
UNIVERSITY

Academic Coaches Conference

Senior Math

A Program of the Indiana Association of School Principals

I. Numeration Systems – 12%
   A. Binary (base 2) and Sexagesimal (base 60) Systems
   B. Convert to and from base 10
   C. Add and subtract in base 2 and base 60

- Binary numbers are based on the powers of 2:

$2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, …

- Binary numbers use only two digits: 0 and 1
- Binary numbers are written as, for example: 11011101

$11011101 = 1x2^7 + 1x2^6 + 0x2^5 + 1x2^4 + 1x2^3 + 1x2^2 + 0x2^1 + 1x2^0 = 221$ (base 10)

- Sexagesimal numbers are based on powers of 60:

$60^0 = 1$, $60^1 = 60$, $60^2 = 3600$ 603 = 216,000, …..

- Sexagesimal numbers need 60 "digits". The digits are

0, 1, 2, ….10, 11, 12, …..57, 58, and 59

Notice that 10, 11, 12, ….57, 58, 59 are single digits, not two digit numbers

- Sexagesimal numbers will be written using commas to separate the digits

For example: 11, 0, 7, 53

$11, 0, 7, 53 = 11x60^3 + 0x60^2 + 7x60^1 + 53x60^0 = 2{,}376{,}473$ (base 10)

Convert to and from base 10

Ex: If the base ten number 55 is written as a binary number, what is the result?

A. 111111
B. 111101
C. 111011
D. 110111 Answer: D

Ex: If the sexagesimal number 1, 12, 47, 0 is converted to base ten, what is the result?

A. 262,020
B. 264,040
C. 266,060
D. 268,080

Answer: A

Ex: If the binary number 1abc10 equals 54 in base 10, what is the SUM of a, b, and c?

A. 0
B. 1
C. 2
D. 3

Answer: C

Ex:
If the binary number 11110101 is converted to the sexagesimal number A, B, what is A + B?

A. 9
B. 10
C. 11
D. 12

Answer: A

Add and subtract in base 2 and base 60

Ex:

What is the sum of the binary numbers 111, 1011, 1101, and 10101?

A. 101110
B. 110100
C. 110010
D. 111100

Answer: B

Ex:

What is the difference, if the sexagesimal number 8, 53, 19 is subtracted from the sexagesimal number 10, 44, 8?

A. 1, 51, 49
B. 1, 51, 59
C. 1, 50, 49
D. 1, 50, 59

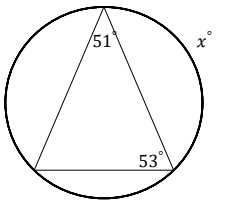
Answer: C

II. Geometry – 28%
- A. Circles
  1. Sectors and segments
  2. Angle measure and arc lengths
  3. Equations of circles
- B. Right Triangles
- C. Lateral area and volume of right solids
- D. Volume of the frustum of square pyramid

A. Circles

Ex: Using the diagram, find the value of x.

A. 76
B. 106
C. 114
D. 152



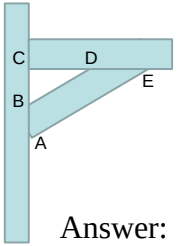
Answer: D

B. Right Triangles

All problems will involve right triangles or combinations of right triangles. Lengths of legs and the hypotenuse only. No questions on area.

Ex:
Cameron made a mailbox post that consisted of a horizontal post, a vertical arm, and a slanted brace as shown. Some of the measurements in inches are CD = 6, DE = 5, CB = 8, and BA = 6 2/3. What is the TOTAL length, BD + AE, of the two edges of the base?

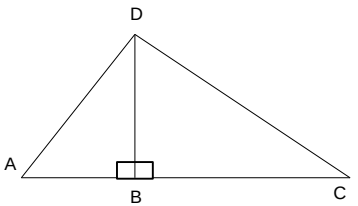
A. 27 2/3
B. 28 1/3
C. 28 2/3
D. 29 1/3



Answer: B

Ex:
In the figure, AC = 6 and AD = BC = 4. Find DC.

A. $\sqrt{22}$
B. $\sqrt{24}$
C. $\sqrt{26}$
D. $\sqrt{28}$



Answer: D

II. C. Lateral area, volume, and lengths in right solids

- Cylinders, cones, prisms, and pyramids only
- For prisms and pyramids, the bases will be equilateral triangles, squares and regular hexagons
- Lengths in solids can be: radius, diameter, circumference, perimeters, apothem or edge of the base, and the height, slant height, or lateral edge.

(There are a lot of right triangle connections between these lengths)

Ex:

Find the lateral area of a cone if the area of the base is 36π and the height is 8.

A. 48π
B. 52π
C. 56π
D. 60π

Answer: D

Ex:

A hexagonal pyramid has a base edge of 6 and a slant height of $5\sqrt{3}$. What is the volume of this pyramid?

A. 216
B. 220
C. 224
D. 228

Answer: A

II. D. Volume of the frustum of a square pyramid

Ex:

The frustum of a square pyramid has a height of 9. The area of the lower base is 49 and the area of the upper base is 36. What is the volume of this frustum?

A. 380
B. 381
C. 382
D. 383

Answer: B

Ex:

The frustum of a square pyramid has a volume of 917.5. The areas of the lower and upper bases are 81 and 169. What is the height of the frustum?

A. 7.25
B. 7.5
C. 7.75
D. 8.00

Answer: B

III. Trigonometry

A. Basic functions
B. Solving right and oblique triangles
C. Simplifying trig expressions using identities (basic and double-angle)
D. Solving trig equations

A. Basic functions

Ex: If $\tan\theta = 2$ and θ is a first quadrant angle, then $\sec\theta =$

A. $\sqrt{5}$

B. $\frac{\sqrt{5}}{2}$

C. $\frac{\sqrt{5}}{5}$

D. $\frac{2\sqrt{5}}{5}$

Answer: A

B. Solving right and oblique triangles

Ex: An airplane pilot wants to clear a 110 foot hill by at least 50 feet. If that pilot starts at a point 1,200 feet from the foot of the hill, what can be the least angle of climb?

A. 6°

B. 7°

C. 8°

D. 9°

Answer: C

- Ex: An isosceles triangle has sides of lengths 20 inches, 20 inches, and 26 inches. What is the measure of the largest angle?

A. 87°

B. 81°

C. 76°

D. 74°

Answer: B

C. Simplifying trig expressions

Know these identities:

Reciprocal

$\csc x = \frac{1}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \cot x = \frac{1}{\tan x}$

Quotient

$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$

Pythagorean

$sin^2x + cos^2x = 1 \quad tan^2x + 1 = sec^2x$

$1 + cot^2x = csc^2x$

Double Angle (sin and cos only)

Ex: Simplify: $\frac{1}{\sec\theta - \tan\theta}$

A. $\cos\theta$

B. $\sec\theta$

C. $\frac{\cos\theta}{1-\sin\theta}$

D. $\cot\theta$

Answer: C

D. Solving trig equations

Ex: Solve $2\cos x + \sin 2x = 0$. The sum of the solutions $(0 < x \leq \pi)$ is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. $\frac{5}{4}\pi$

Answer: B

IV. Polynomials and Factoring

A. Polynomial expressions (all operations)

B. Factoring expressions

1. Standard methods of factoring (2nd & 3rd degree)
2. Factoring via synthetic division
3. Finding zeroes of polynomial functions
4. End behavior of graphs of polynomial functions

Ex: How many possible rational zeros are there for the function $f(x) = 6x^3 + 7x^2 + 3x - 4$?

A. 12
B. 16
C. 18
D. 24

Answer: B

V. History – 16%

- All history topics will come from material on the website, MacTutor History Topics Index

Specifically –

A. Ancient Babylonian Mathematics
   1. An overview of Babylonian mathematics
   2. Babylonian numerals
   3. Pythagoras's theorem in Babylonian mathematics

B. Ancient Egyptian Mathematics
   1. An overview of Egyptian mathematics
   2. Mathematics in Egyptian papyri
   3. Egyptian numerals

- Students must have a thorough knowledge of these 6 documents.

- To access the previously mentioned documents:

Google: MacTutor History of Mathematics
Click on History Topics Index
Click on Ancient Babylonian mathematics
Print the three documents listed on previous page

Likewise, click on Ancient Egyptian mathematics
Print the three documents listed on previous page

V.A. 2 Babylonian Numerals

- The Babylonians had a base 60, positional number system.
- They therefore needed 59 symbols for the "digits" of their system. They did not have a symbol for 0, but they were able to write their 59 digits just using combinations of two symbols,  for 1 and  for 10.

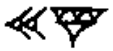
So,  was the digit 24.

In the MacTutor document there is a table of all 59 digits.

Can you guess what this is 

V.A. 2 Babylonian Numerals

- However, there are two problems: The Babylonians had no way to indicate 0 and they didn't separate different digits in the same number very well, if at all.

So,  could be the digit 24,

Or the two digit number: $20x60 + 4 = 1,204$

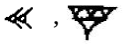
Or the three digit number: $20x60^2 + 0x60 + 4 = 72,004$

Or the three digit number: $10x60^2 + 14x60 + 0 = 36,840$

To solve this problem I will put commas between the Babylonian digits.

24 will be 

72,004 will be 

1,204 will be 

36,840 will be 

Ex:

If the Babylonian number  is written in base 10, what would it be?

A. 2,170,831
B. 2,543,261
C. 2,811,741
D. 3,042,511

Answer: A

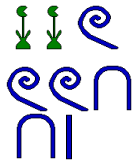
V.B. 3: Egyptian Numerals

The Egyptians had a simple hieroglyphic number system in which a single symbol stood for 1, another for 10, another for 100, etc.

I will use the following four symbols:

= 1 The symbol for 1 may come from a finger
= 10 The symbol for 100 is a piece of rope
= 100 The symbol for 100 is a coil of rope
= 1000 The symbol for 1000 is the lotus or water lily

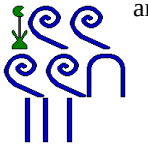
(The Egyptians had symbols for 10^4, 10^5, and 10^6 as well, but I will not use those.)

So,  is the Egyptian hieroglyphic for writing 2321.

They simply put as many of each symbol as they needed to make the number.

This is obviously a primitive and unwieldy way to write numbers.

However, they are easy to add and subtract.

Ex: If  and  are added,

what would the result be?

A. 3023
B. 2923
C. 2823
D. 2723

Answer: A

- After thousands of years and the invention of the reed paper papyrus, the Egyptians started using a new system for numbers now called the hieratic system. This system had many more symbols, so numbers could be written in a much more compact way than with the hieroglyphic method.
- Students should know about the hieratic system, but they DO NOT need to know the individual symbols. I did not write any questions using the hieratic symbols.
- Finally, the history topics discuss how both the Babylonians and the Egyptians were able to write fractions. Students should know that they could do this, but I DID NOT write any questions involving fractions.

- For the rest of the history topics I will just give you two examples:

Ex:

Which is correct about the Egyptian hieroglyphic and the Babylonian sexagesimal systems and the ability to write fractions?

A. The Egyptians could write fractions, but the Babylonians could not
B. The Babylonians could write fractions, but the Egyptians could not
C. Both cultures could write fractions
D. Neither culture could write fractions

Answer: C

Ex:

Strong evidence has been found that one of the cultures, Egyptian or Babylonian, had a method for finding the volume of a square frustum. On which of the following was this evidence found?

A. Yale tablet
B. Plimpton 322
C. Rhind papyrus
D. Moscow papyrus

Answer: D

REMEMBER: Only the TI-30XA and the TI-30XIIS may be used during competitions!

There are no exceptions!

For questions or comments on
Numeration Systems, Right Triangles,
Lateral area, volume, and lengths in right solids
Volume of a frustrum of a square pyramid
History
contact Danny Dixon at dndixon@comcast.net

For questions or comments on
Circles, Trigonometry, and Polynomials and Factoring
contact Garett Cates at g.cates@comcast.net